Kyle Academy

Physics Department

CfE Higher Physics

 Significant Figures & Uncertainties

Name _____________________________

Cultivating Excellence in Science
Significant Figures

It is important when calculating numerical values that the final answer is quoted to an appropriate number of significant figures. As a general rule, the final numerical answer that you quote should be to the same number of significant figures as the data given in the question.

The above rule is the key point but you might like to note the following points:

1. The answer to a calculation cannot increase the number of significant figures that you can quote.

2. If the data is not all given to the same number of significant figures, identify the least number of significant figures quoted in the data. This least number is the number of significant figures that your answer should be quoted to.

3. When carrying out sequential calculations carry many significant figures as you work through the calculations. At the end of the calculation, round the answer to an appropriate number of significant figures.

4. In the Higher Physics course quoting an answer to three significant figures will usually be acceptable.

Examples

1. The current in a circuit is 6.7 A and the voltage across the circuit is 21 V. Calculate the resistance of the circuit.

Note: Both of these pieces of data are given to two sig. figs. so your answer must also be given to two sig figs.

\[ I = 6.7 \, \text{A} \]
\[ V = 21 \, \text{V} \]
\[ R = ? \]

2. A 5.7 kg mass accelerates at 4.36 m s\(^{-2}\). Calculate the unbalanced force acting on the mass.

Note: The mass is quoted to two sig. figs and the acceleration is quoted to three sig. figs. so the answer should be quoted to two sig figs.

\[ m = 5.7 \, \text{kg} \]
\[ a = 4.36 \, \text{m \, s}^{-2} \]
\[ F = ? \]
3. A car accelerates from \(0.5037\ \text{m s}^{-1}\) to \(1.274\ \text{m s}^{-1}\) in a time of \(4.25\ \text{s}\). The mass of the car is \(0.2607\ \text{kg}\). Calculate the unbalanced force acting on the car.

Note: The time has the least number of sig figs, three, so the answer should be quoted to three sig figs.

\[u = 0.5037\ \text{m s}^{-1}\]
\[v = 1.274\ \text{m s}^{-1}\]
\[t = 4.25\ \text{s}\]
\[m = 0.2607\ \text{kg}\]

Quiz questions

Q1: A car travels a distance of \(12\ \text{m}\) in a time of \(9.0\ \text{s}\). The average speed of the car is:

a) 1.3333
b) 1.33
c) 1.3
d) 1.4
e) 1

Q2: A mass of \(2.26\ \text{kg}\) is lifted a height of \(1.75\ \text{m}\). The acceleration due to gravity is \(9.8\ \text{m s}^{-2}\). The potential energy gained by the mass is:

a) 38.759 J
b) 38.76 J
c) 38.8 J
d) 39 J
e) 40 J

Q3: A trolley of \(5.034\ \text{kg}\) is moving at a velocity of \(4.03\ \text{m s}^{-1}\). The kinetic energy of the trolley is:

a) 40.878 J
b) 40.88 J
c) 40.9 J
d) 41 J
e) 40 J
Prefixes for Higher Physics

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>pico</td>
<td>$p$</td>
<td>$x \times 10^{-12}$</td>
</tr>
<tr>
<td>nano</td>
<td>$n$</td>
<td>$x \times 10^{-9}$</td>
</tr>
<tr>
<td>micro</td>
<td>$\mu$</td>
<td>$x \times 10^{-6}$</td>
</tr>
<tr>
<td>milli</td>
<td>$m$</td>
<td>$x \times 10^{-3}$</td>
</tr>
<tr>
<td>kilo</td>
<td>$k$</td>
<td>$x \times 10^{3}$</td>
</tr>
<tr>
<td>mega</td>
<td>$M$</td>
<td>$x \times 10^{6}$</td>
</tr>
<tr>
<td>giga</td>
<td>$G$</td>
<td>$x \times 10^{9}$</td>
</tr>
<tr>
<td>tera</td>
<td>$T$</td>
<td>$x \times 10^{12}$</td>
</tr>
</tbody>
</table>

Uncertainties

1) **Absolute uncertainties and Percentage uncertainties**

Every time measurement is made it will have an uncertainty in it.

If a reading is made using an ammeter to record a value of 1.5 A, there is a *scale-reading uncertainty* of $\pm 0.1$ A in the last digit. The reading on the meter is therefore $1.5 \pm 0.1$ A. When written in the form (value $\pm$ uncertainty) this is called the **absolute uncertainty**.

The **percentage uncertainty** in this reading is $(0.1/1.5) \times 100 = 6.7\%$

Example 1:

A student record a measurement as 5.5 cm with an uncertainty of $\pm 0.2$ cm.

State the

i) absolute value

ii) value with a percentage uncertainty.

Example 2:

A student record a measurement as 30.55 $\Omega$.

State the

i) absolute value

ii) value with a percentage uncertainty.
2) **Random uncertainties**

When an experiment is repeated several times, the result may not be the same each time with some values slightly higher or lower than the true value. Examples such as using a stop-watch, measuring an angle with a protractor or using a measuring tape.

The effects of random uncertainties can be reduced by repeating measurements and finding the mean value. The mean value is the best estimate of the true value being measured.

Relationship for random uncertainties:

\[
\text{random uncertainty} = \frac{\text{max. value} - \text{min. value}}{\text{number of values}}
\]

**Notes:**

1. A random uncertainty can only be calculated from measured data that you would expect to be the same value.
2. A random uncertainty must not be found in calculated values.

**Example:** A student uses a computer program to measure their reaction time. The following values are obtained for the reaction time of the student.

<table>
<thead>
<tr>
<th>Attempt number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaction time /s</td>
<td>0.273</td>
<td>0.253</td>
<td>0.268</td>
<td>0.273</td>
<td>0.238</td>
</tr>
</tbody>
</table>

a) Calculate the mean reaction time of the student.

b) Calculate the approximate random uncertainty in the mean.

**Activity 1:** The teacher will release a ball from the same height 5 times. Record the time taken for the ball to reach the floor. Calculate the mean and the random uncertainty in the mean. (Watch for significant figures in the mean value and in the random uncertainty!)

**Activity 2:** Using a trolley and ramp find the time taken for the trolley to run down the ramp. Find the mean value and the random uncertainty in the mean.
**Increasing the reliability**

In order to increase the reliability of a measurement, increase the number of times that the quantity is measured. It is likely that the random uncertainty will decrease.

**Activity 3:** Repeat Activity 2 using the same trolley and ramp but this time take 10 readings and then find the mean with the random uncertainty.

What is the effect of an increased number of readings on the random uncertainty?

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3) **Scale-reading uncertainties**

A scale-reading uncertainty is a measure of how well an instrument scale can be read. For analogue scales, the scale-reading uncertainty is usually taken as ± half of the smallest scale division. For digital scales the reading uncertainty is ± 1 in the last (least significant) digit.

1. **Example 1: Analogue scale**

This approach is used for rulers, metre sticks, liquid in glass thermometer and meters which have a pointer.

The length of metal is measured with the ruler shown below.

![Ruler image]

The length is

Scale reading uncertainty is

Value expressed as

True value is between and
2. Example 2: Digital display
This approach is used whenever a seven segment digital display is present.
The image below shows a digital ammeter.

![Digital Ammeter](image)

Current =
Scale reading uncertainty =
Expressed as
True value is between and

4) **Systematic uncertainties**

Systematic uncertainties have consistent effects on the quantities being measured. Systematic uncertainties often arise due to experimental design or issues with the equipment. Readings taken are either all too small or all too large.

The ruler below shows an example of a systematic error.

![Ruler Example](image)

It looks at first to show the metal bar as 8 cm but on closer inspection it is just 7 cm. The ruler starts at 1 cm and not 0 cm. All values measured using this ruler would be 1 cm too long. This would be a systematic error. A systematic error can be corrected by the experimenter.

If a graph from an experiment is a straight line and does not pass through the origin, you should consider the possibility that you have a systematic uncertainty.
Quiz

Q1: State the scale reading uncertainty in the following voltmeter reading.

![](voltmeter.png)

a) ± 0.25 V  
b) ± 0.5 V  
c) ± 1.0 V  
d) ± 2.0 V  
e) ± 5.5 V

Q2: A student carries out an investigation to measure the time taken for ten complete swings of a pendulum. The following values are obtained for the time for ten complete swings.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>3.1 s</th>
<th>3.8 s</th>
<th>3.3 s</th>
<th>4.1 s</th>
<th>3.4 s</th>
</tr>
</thead>
</table>

a) ± 0.01 s  
b) ± 0.02 s  
c) ± 0.1 s  
d) ± 0.2 s  
e) ± 1.0 s

Q3: A student carries out three investigations into the variation of voltage and current. The results obtained are shown in the Graphs A, B and C.

![](graphA.png)  
![](graphB.png)
Which of the following statements is/are true?

- I Graph A shows a systematic uncertainty
- II Graph B shows a proportional relationship
- III Graph C shows a systematic uncertainty

a) I only  
b) II only  
c) I and II only  
d) I and III only  
e) I, II and III

Q4: In an experiment the following measurements and uncertainties are recorded.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature rise</td>
<td>10°C ± 1°C</td>
</tr>
<tr>
<td>Heater current</td>
<td>5.0 A ± 0.2 A</td>
</tr>
<tr>
<td>Heater voltage</td>
<td>12.0 V ± 0.5 V</td>
</tr>
<tr>
<td>Time</td>
<td>100 s ± 2 s</td>
</tr>
<tr>
<td>Mass of liquid</td>
<td>1.000 kg ± 0.005 kg</td>
</tr>
</tbody>
</table>

The measurement which has the largest percentage uncertainty is the:

a) Temperature rise  
b) Heater current  
c) Heater voltage  
d) Time  
e) Mass of liquid

Q5: In an investigation the acceleration of a trolley down a slope is found to be 2.5 ms\(^{-2}\) ± 4%.

The absolute uncertainty in this value of acceleration is:

a) ± 0.04 m s\(^{-2}\)  
b) ± 0.1 m s\(^{-2}\)  
c) ± 0.4 m s\(^{-2}\)  
d) ± 1.0 m s\(^{-2}\)  
e) ± 4.0 m s\(^{-2}\)
When an experiment is being undertaken and more than one physical quantity is measured, the quantity with the largest percentage uncertainty should be identified and this may often be used as a good estimate of the percentage uncertainty in the final numerical result of an experiment. The numerical result of an experiment should be expressed in the form final value ± uncertainty.

Q6: In an investigation the voltage across a resistor is measured as 20 V ± 2 V and the current through it is 5.0 A ± 0.1 A. The percentage uncertainty in the power is:

a) 0.1%
b) 2%
c) 3%
d) 10%
e) 12%

Q7: Specific heat capacity can be found from the experimental results given below. Which one of the following measurements creates most uncertainty in the calculated value of the specific heat capacity?

a) Power = 2000 ± 10 W
b) Time = 300 ± 1 s
c) Mass = 5.0 ± 0.2 kg
d) Final temperature = 50 ± 0.5°C
e) Change in temperature = 30 ± 1°C

Q8: The light coming from a spectral lamp is investigated and the following data is obtained.

\[ \lambda = 450 \text{ nm} \pm 10\% \]
\[ f = 6.7 \times 10^{14} \text{ Hz} \pm 2\% \]

This data is used to estimate the speed of light. The absolute uncertainty in this estimate of the speed of light is:

a) ± 2.0 m s\(^{-1}\)
b) ± 10 m s\(^{-1}\)
c) ± 6.0 \times 10^6 \text{ m s}^{-1}\)
d) ± 3.0 \times 10^7 \text{ m s}^{-1}\)
e) ± 3.6 \times 10^7 \text{ m s}^{-1}\)
Q9: Two forces $P$ and $Q$ act on an object $X$ as shown.

\[ P \ 16.35 \text{ N} \pm 0.02 \text{ N} \quad \text{X} \quad Q \ 1.87 \text{ N} \pm 0.01 \text{ N} \]

The value of the unbalanced force acting on the object $X$ and the percentage uncertainty in this value, expressed in the form value $\pm$ absolute uncertainty is:

a) $14.48 \text{ N} \pm 0.03 \text{ N}$

b) $14.48 \text{ N} \pm 0.08 \text{ N}$

c) $14.48 \text{ N} \pm 0.5 \text{ N}$

d) $18.22 \text{ N} \pm 0.03 \text{ N}$

e) $18.22 \text{ N} \pm 0.08 \text{ N}$

Q10: A student measures their reaction time using the digital stop watch on a computer. The following measurements of their reaction time are displayed on the computer's digital stop watch.

| 0.29 s | 0.25 s | 0.22 s | 0.26 s | 0.24 s |

When evaluating this set of measurements the student makes the following statements.

- I Increasing the number of attempts from 5 to 10 would make the mean value more reliable.
- II The scale reading uncertainty in this set of measurements is $\pm 0.01 \text{ s}$. 
- III You can tell by reviewing the measurements that there is no systematic uncertainty present.

Which of the above statements is/are correct?

a) I only

b) II only

c) III only

d) I and II only

e) I, II and III
Past Paper Questions on uncertainties

1. Two students are investigating reaction time. Student A holds a ruler vertically between the fingers of student B as shown.

![Diagram of students A and B with a ruler]

Student A then releases the ruler from rest and student B catches it as quickly as she can. The distance that the ruler falls before being caught is measured. The experiment is repeated a number of times and the following distances recorded.

\[ 0.164 \text{ m} \quad 0.190 \text{ m} \quad 0.188 \text{ m} \quad 0.155 \text{ m} \quad 0.163 \text{ m} \]

(a) Calculate:

(i) the mean value of the distance fallen;  
(ii) the approximate random uncertainty in this value.

(b) Use the mean value of the distance fallen to calculate the reaction time of student B. The uncertainty in the reaction time is not required.
2. An experiment is carried out to measure the time taken for a steel ball to fall vertically through a fixed distance using an electronic timer.

(a) The experiment is repeated and the following values for time recorded.

0.49 s, 0.53 s, 0.50 s, 0.50 s, 0.55 s, 0.51 s.

Calculate:

(i) the mean value of the time; 1
(ii) the approximate random uncertainty in the mean value of the time. 1

3. Distance AB is measured six times.

The results are shown.

\[ 1.11 \text{ m}, 1.08 \text{ m}, 1.10 \text{ m}, 1.13 \text{ m}, 1.11 \text{ m}, 1.07 \text{ m} \]

(i) Calculate:

(A) the mean value for distance AB; 1
(B) the approximate random uncertainty in this value. 1

(ii) Distance BC is measured as \((270 \pm 10) \text{ mm} \).

Show whether AB or BC has the larger percentage uncertainty. 2

4. A ball is dropped several times from the same height.

A student records the following times for the ball to reach the ground.

\[ 1.15 \text{ s}, 1.13 \text{ s}, 1.09 \text{ s}, 1.13 \text{ s}, 1.05 \text{ s} \]

Which row in the table shows the mean time for the ball to reach the ground and the approximate random uncertainty in this mean?

<table>
<thead>
<tr>
<th>Mean time/s</th>
<th>Approximate random uncertainty/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.11</td>
</tr>
<tr>
<td>B</td>
<td>1.11</td>
</tr>
<tr>
<td>C</td>
<td>1.13</td>
</tr>
<tr>
<td>D</td>
<td>1.13</td>
</tr>
<tr>
<td>E</td>
<td>4.71</td>
</tr>
</tbody>
</table>

Collect a copy of the Progressive Problems text book and complete questions 1-6 on pages 142/143.

Complete all the questions on page 132 of the How to Pass Higher Physics text book.